Stueckelberg mechanism and chiral Lagrangian for Z' boson

Ying Zhang

School of Science, Xi'an Jiaotong University,
Xi'an, 710049, P.R.China
E-mail: `hepzhy@mail.xjtu.edu.cn`

Shun-Zhi Wang

Department of Physics, Tsinghua University,
Beijing, 100084, P.R.China
E-mail: `wsz04@mails.tsinghua.edu.cn`

Qing Wang

Center for High Energy Physics, Tsinghua University,
Beijing 100084, P.R.China
E-mail: `wangq@mail.tsinghua.edu.cn`

ABSTRACT: Traditional Stueckelberg Mechanism is shown equivalent to set up a gauged U(1) chiral Lagrangian and fix special gauge. With this mechanism, the original electro-weak chiral Lagrangian is inlarged by including an extra U(1) symmetry to represent physics for Z' boson. We build up complete list of electro-weak chiral Lagrangian up to order of p^4 including Z' and higgs bosons. The most general mixing among neutral gauge bosons is diagonalized completely and the connections among these operators to triple, quartic couplings involving Z' boson and to that in traditional electro-weak chiral Lagrangian are made.

Contents

1. Introduction

Seventy years before, C.G. Stueckelberg [1] introduced a scalar into massive abelian vector theory without violation of gauge symmetry and renormalizability. Since then, people used to apply this mechanism to describe massive photon. Beyond that, many other applications also emerged, such as those to SM [2, 3], MSSM [4], string [5] and extra dimension [6], etc. The latest review was given by ref. [7]. This mechanism in literature was seen as a scheme to replace Higgs mechanism for broken U(1) gauge theory [8] in the sense that it does not need Higgs particle. Among various applications, we are interested in this work in investigating physics of Z' boson. On the one hand, as a heavy undiscovered new vector particle in the minimal extension of SM, Z' will probably be the particle easest to test in future collider experiments and plays important role in various new physics models, such as low energy models induced from GUT and SUSY [9 – 12], left-right symmetric models [9], little Higgs models [13] and extra dimension models [14, 15], etc; on the other hand, Stueckelberg mechanism provides us a special method to introduce abelian massive vector into theory gauge invariantly. With this mechanism, we can simply add Z' boson to SM and discuss corresponding physics [2]. However, the traditional Stueckelberg mechanism only deals with lowest dimension term related to vector boson mass and leads typical mixing term between scalar particle and gauge boson, which does not include those more complex high dimension operators. As a consequence, this approach lost generality in the sense that operator involving Z' boson through Stueckelberg mechanism is that with lowest dimension which represents a special kind of Z' interaction. Though this operator plays the most important role in low energy region, it is not general enough when we approach to TeV energy region where effects of high dimension operators will emerge. These high dimensional operators, most of them are non-renormalizable, are effective description of

underlying new physics dynamics. Adding in these non-renormalizable high dimension operators into theory is a necessary step when we want to go beyond SM to investigate new physics model independently. This requirement leads to the generalization of the traditional Stueckelberg mechanism by including high dimension operators into theory so that general Z' interactions may be covered as much as possible. With non-renormalizable operators included in, renormalizability of original theory is lost and is replaced by a generalized version of renormalization for effective field theory [16].

There are two ways to systematically describe general effective interactions among particles in SM: namely, linear and nonlinear realizations of SM symmetry $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em}$. Within linear realization, we just add in high dimension operators into SM [17–20]. While in nonlinear realization, we start from electroweak chiral Lagrangian (EWCL) [21–23] which is the most general description for SM fields except Higgs. This EWCL was generalized to extended electroweak chiral Lagrangian (EEWCL) by adding in original EWCL a singlet Higgs field [24] to keep unitarity of the theory [25]. Though mathematical equivalence between two descriptions was shown in [24], linear realization is suitable for discussion of light Higgs, while nonlinear realization can be applied to investigate either light or heavy Higgs. Due to this generality for EEWCL, we use nonlinear realization in this paper. In fact, we will show that Stueckelberg mechanism is equivalent to chiral Lagrangian for $U(1)$ gauge field plus special choice of gauge fixing term. This equivalence enable us to further understand the non-renormalizability for Stueckelberg mechanism when we try to generalize it to non-abelian gauge field system and base our whole discussion on the non-linear realization of SM symmetry. With the equivalence of Stueckelberg mechanism and $U(1)$ chiral Lagrangian, the generalization of traditional Stueckelberg mechanism become obvious: we just extend EEWCL with an extra $U(1)$ gauge symmetry and write down all possible high dimension interaction terms. To make particle content in our discussion close to low energy particle spectrum already discovered in experiment, except Higgs and Z' bosons, we do not involve any other new undiscovered particles in our theory. Higgs particle in this work only plays a passive role and we mainly focus our attention on Z' interactions.

This paper is organized as follows. Section 2 is the proof for the equivalence of traditional Stueckelberg mechanism and $U(1)$ chiral Lagrangian and discussion of its nonabelian generalization. In section 3, we generalize original EEWCL to include Z' boson and write down the bosonic part of Lagrangian up to order of p^4. From this Lagrangian, we obtain the most general mixing for neutral gauge bosons. Then we completely diagonalize and discuss the mixing. In section 4, We build up the connections of these operators to triple , quartic couplings involving Z' boson and traditional electro-weak chiral Lagrangian. The summary is given in section 4.

2. Equivalence between Stueckelberg mechanism and chiral Lagrangian

Now, let us review Stueckelberg mechanism. The most simple Stueckelberg Lagrangian for massive vector A_μ can be written as

$$\mathcal{L}_{\text{Stueck}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}\left(A_\mu - \frac{1}{m}\partial_\mu\sigma\right)^2 , \qquad (2.1)$$

with obvious mass term $m^2 A_\mu^2/2$. Under U(1) gauge transformation $A_\mu \to A_\mu + \partial_\mu \epsilon$, $\sigma \to \sigma + m\epsilon$, the Lagrangian is invariant. Adding a gauge fixing term

$$\mathcal{L}_{\text{GF}} = -\frac{1}{2\xi}(\partial_\mu A^\mu + \xi m\sigma)^2 \tag{2.2}$$

into the Lagrangian, the total Lagrangian is the sum of Stueckelberg Lagrangian $\mathcal{L}_{\text{Stueck}}$ and gauge fixing term $\mathcal{L}_{\text{GF}}$

$$\mathcal{L}_{\text{total}} = -\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{\xi}{2}m^2\sigma^2. \tag{2.3}$$

Mixing term $\sigma\partial_\mu A^\mu$ appeared in $\mathcal{L}_{\text{GF}}$ cancels the same term in $\mathcal{L}_{\text{Stueck}}$. This leads to the decoupling of auxiliary scalar σ and vector field A_μ. The unphysical σ is given a mass proportional to random parameter $\sqrt{\xi}$, which means σ is unphysical field and have no any influence on vector field A_μ. So traditional Stueckelberg mechanism include two parts. One is extension of standard mass term of U(1) gauge boson through term mixing with differential of scalar field. This part, we will show, is equivalent to gauged U(1) chiral Lagrangian. The other is choice of special gauge fixing term to cancel mixing between scalar and gauge boson.

Now we prove the assertion that the first part of traditional Stueckelberg mechanism is equivalent to gauged U(1) chiral Lagrangian. We change σ field by introducing an unitary phase angle field U as

$$U(x) \equiv e^{i\frac{\sigma(x)}{m}} . \tag{2.4}$$

Under U(1) gauge transformation, it transforms as $U \to e^{i\epsilon}U$. We can construct covariant derivative for U as

$$D_\mu U(x) \equiv [\partial_\mu - iA_\mu(x)] U(x) = i U(x)\left[\frac{1}{m}[\partial_\mu \sigma(x)] - A_\mu(x)\right] . \tag{2.5}$$

With this covariant derivative, we can rewrite (2.1) in terms of U field as

$$\mathcal{L}_{\text{Stueck}} = -\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}(D^\mu U)^\dagger (D_\mu U) , \tag{2.6}$$

which is standard lowest p^2 order chiral Lagrangian (gauged nonlinear σ model) for U(1) gauge field as long as we identify m with goldstone decay constant f. Here σ plays the role of goldstone boson which, in terms of Higgs mechanism, will be eaten out by gauge field A_μ to become its longitudinal part after symmetry breaking. Broken U(1) symmetry is explicitly seen through unitary gauge $U = 1$ (or taking vacuum).

In terms of our U field representation, gauge fixing term (2.2) can be written as

$$\mathcal{L}_{\text{GF}} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu - i\xi m^2 \ln U\right)^2 , \tag{2.7}$$

which can cancel the mixing term between A_μ and σ and make σ becoming free field.

Above equivalence between Stueckelberg mechanism and gauged U(1) chiral Lagrangian can be seen as an alternative statement for the distinction of the Stueckelberg and the Higgs mechanisms for which conventional understanding relies on the existance of a Higgs particle [4]. Now chiral Lagrangian is a formalism constructed by gauge field and corresponding goldstone boson, it does not need Higgs field and therefore in this sense is the same as Stueckelberg mechanism. In fact, this equivalence was pointed out in an alternative way in ref. [26]. With this equivalence, applications of Stueckelberg mechanism can be realized in terms of standard formalism of chiral Lagrangian. One possible application is to consider effects from high dimension operators which as mentioned in last section may reflect more complex and general interactions among Z' boson and SM particles. This will be discussed in next section. Another direction of application is to generalize U(1) to nonabelian gauge symmetry. In the following part of this section, we take a simplest nonabelian generalization by considering following symmetry breaking realization $SU(2)_L \otimes SU(2)_R \to SU(2)_D$ with 2×2 unitary matrix field $\tilde{U}$ defined as

$$\tilde{U}(x) \equiv e^{\frac{i}{m}\tilde{\sigma}_i(x)\tau_i} = m \left[\sqrt{1 - \frac{\Sigma^2(x)}{m^2}} + i\frac{\Sigma_i(x)\tau_i}{m} \right] \quad \Sigma_i(x) \equiv \frac{m\tilde{\sigma}_i(x)}{\sqrt{\tilde{\sigma}^2(x)}} \sin\frac{\sqrt{\tilde{\sigma}^2(x)}}{m} \, , \quad (2.8)$$

where τ_i, $i = 1, 2, 3$ are Pauli matrices, and $\tilde{\sigma}_i$ are three goldstone bosons generated from $SU(2)_L \otimes SU(2)_R \to SU(2)_D$ through Goldstone theorem. The $SU(2)_L \otimes SU(2)_R$ gauge transformation is

$$\tilde{U}(x) \to \tilde{V}_R(x)\tilde{U}(x)\tilde{V}_L^\dagger(x) \tag{2.9}$$

in which $\tilde{V}_R$ and $\tilde{V}_L$ are $SU(2)_R$ and $SU(2)_L$ group elements respectively.

Note that if we return back from (2.9) to our original abelian situation, U field will transform as

$$U(x) \to V_R(x) U(x) V_L^\dagger(x) = V_R(x)V_L^\dagger(x) U(x) \, , \tag{2.10}$$

where $V_L = e^{i\epsilon_L}$ and $V_R = e^{i\epsilon_R}$ is $U(1)_L$ and $U(1)_R$ group element, respectively. Consider $U(1)_L \otimes U(1)_R = U(1)_D \otimes U(1)$, with $V_D = e^{i(\epsilon_R + \epsilon_L)}$ and $V_A = e^{i(\epsilon_R - \epsilon_L)} = V_R V_L^\dagger$ being corresponding $U(1)_D$ and $U(1)$ group elements respectively. From (2.10), it is easy to see that U field is invariant under $U(1)_D$ transformation and therefore $U(1)_D$ is a trivial symmetry for Lagrangian (2.6). With this trivial $U(1)_D$ symmetry included in (2.6), the symmetry realization pattern for original Stueckelberg Lagrangian become $U(1)_L \otimes U_R(1) \to U(1)_D$. With this form of ablelian symmetry realization for original Stueckelberg Lagrangian, our nonabelian generalization of $SU(2)_L \otimes SU(2)_R \to SU(2)_D$ become obvious. The only difference from abelian case is that the left unbroken symmetry $SU(2)_D$ is not a trivial symmetry in the sense that $\tilde{U}$ is not invariant under its transformations.

Now we write down the Stueckelberg Lagrangian for $SU(2)_L \otimes SU(2)_R \to SU(2)_D$,

$$\mathcal{L}_{Stueck-SU(2)} = -\frac{1}{4g_L^2}F_{L,i}^{\mu\nu}F_{L,i}^{\mu\nu} - \frac{1}{4g_R^2}F_{R,i}^{\mu\nu}F_{R,i}^{\mu\nu} + \frac{m^2}{4}\text{tr}\left[\left(D^\mu\tilde{U}\right)^\dagger \left(D_\mu\tilde{U}\right) \right] \, , \quad (2.11)$$

with

$$D^\mu \tilde{U} \equiv \partial^\mu \tilde{U} - i\frac{\tau_i}{2}\left(\tilde{V}_i^\mu + \tilde{A}_i^\mu\right)\tilde{U} + i\tilde{U}\frac{\tau_i}{2}\left(\tilde{V}_i^\mu - \tilde{A}_i^\mu\right) \tag{2.12}$$

$$F_{\substack{R,i \\ L,i}}^{\mu\nu}\frac{\tau_i}{2} = \partial^\mu\left(\tilde{V}^\nu \pm \tilde{A}^\nu\right) - \partial^\nu\left(\tilde{V}^\mu \pm \tilde{A}_i^\mu\right) - i\left[\tilde{V}^\mu \pm \tilde{A}^\mu, \tilde{V}^\nu \pm \tilde{A}^\nu\right] \quad \tilde{V}^\mu \equiv \tilde{V}_i^\mu\frac{\tau_i}{2} \quad \tilde{A}^\mu \equiv \tilde{A}_i^\mu\frac{\tau_i}{2} \ ,$$

where $\tilde{V}_i^\mu$, $i = 1, 2, 3$ are $\mathrm{SU}(2)_D$ gauge fields and $\tilde{A}_i^\mu$, $i = 1, 2, 3$ are $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R / \mathrm{SU}(2)_D$ axial gauge fields. In unitary gauge, the third term of r.h.s. of (2.11) becomes mass term $\frac{1}{2}m^2\tilde{A}^2$ of the axial gauge boson field $\tilde{A}$. Due to unbroken symmetry $\mathrm{SU}(2)_D$, corresponding gauge fields $\tilde{V}_i^\mu$, $i = 1, 2, 3$ remain massless.

In terms of fields Σ_i which is already expressed as function of $\tilde{\sigma}_i$ in (2.8), covariant derivative (2.12) now is

$$D^\mu\tilde{U} = \left[-\frac{\Sigma_i}{m\sqrt{1-\frac{\Sigma^2}{m^2}}} + i\frac{\tau_i}{m}\right]\partial^\mu\Sigma_i + \frac{1}{m}\tilde{A}_i^\mu\left[\Sigma_i - i\tau_i\sqrt{1-\frac{\Sigma^2}{m^2}}\right] + \frac{i}{m}\tilde{V}_i^\mu\Sigma_j\epsilon_{ijk}\tau_k. \tag{2.13}$$

With it, (2.11) become

$$\mathcal{L}_{Stueck-\mathrm{SU}(2)} = -\frac{1}{4g_L^2}F_{L,i}^{\mu\nu}F_{L,i}^{\mu\nu} - \frac{1}{4g_R^2}F_{R,i}^{\mu\nu}F_{R,i}^{\mu\nu} \tag{2.14}$$

$$+\frac{1}{2}\left(-\frac{\Sigma_i}{\sqrt{1-\frac{\Sigma^2}{m^2}}}\partial^\mu\Sigma_i + \tilde{A}_i^\mu\Sigma_i\right)\left(-\frac{\Sigma_{i'}}{\sqrt{1-\frac{\Sigma^2}{m^2}}}\partial_\mu\Sigma_{i'} + \tilde{A}_{i',\mu}\Sigma_{i'}\right)$$

$$+\frac{1}{2}\left(\partial^\mu\Sigma_i - \tilde{A}_i^\mu\sqrt{1-\frac{\Sigma^2}{m^2}} + \tilde{V}_j^\mu\Sigma_k\epsilon_{ijk}\right)\left(\partial_\mu\Sigma_i - \tilde{A}_{i,\mu}\sqrt{1-\frac{\Sigma^2}{m^2}} + \tilde{V}_{j'}^\mu\Sigma_{k'}\epsilon_{ij'k'}\right).$$

We find that not only the terms linear in gauge fields $\tilde{V}_i^\mu$ and $\tilde{A}_i^\mu$ mix with Σ_j fields, but the terms bilinear in gauge fields also mix with Σ_j fields which is the general feature for non-abelian gauged nonlinear σ model. This is not like the case of original abelian gauge field, where terms bilinear in gauge fields do not mix with Σ_j fields. This feature makes it impossible to use gauge fixing term to cancel mixing among gauge fields and goldstone fields. Further nonabelian effects cause very complex dependence on goldstone fields which make theory non-renormalizable. This example explicitly shows why generalization of Stueckelberg mechanism to non-abelian case can not cancel mixing among scalars and gauge fields and then cause a coupled non-renormalizable theory.

3. Generalized Stueckelberg mechanism and EEWCL for Z' boson

As mentioned in section 1, nonlinear realized effective field theory EEWCL is already worked out by one of us in ref. [24]. Although this EEWCL only involve boson fields in SM, it's enough for our interests. In this section we are going to generalize it to include in Z' boson. The symmetry realization pattern is then generalized from original $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{em}}$ to $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_{Z'} \to \mathrm{U}(1)_{\mathrm{em}}$. From equivalence between Stueckelberg mechanism and chiral Lagrangian discussed in last section, to apply

generalized Stueckelberg mechanism to Z' boson for EEWCL is equivalent to add into EEWCL a phase degree of freedom representing goldstone boson eaten out by Z' and then gauging in Z' gauge field. We insert this goldstone boson degree of freedom by enlarging original two by two unimodular matrix U field with an extra U(1) phase factor, The new two by two field will be denoted by $\hat{U}$. The difference between U and $\hat{U}$ is that U is unimodular which satisfies constraint det$U = 1$ while $\hat{U}$ does not. Relaxing this unimodular constraint allows an extra U(1) phase in U field which now is identified with mixture of goldstone bosons for Z and Z'. We define the covariant derivative as

$$D_\mu \hat{U} = \partial_\mu \hat{U} + ig W_\mu \hat{U} - i\hat{U}\frac{\tau_3}{2} g' B_\mu - i\hat{U}(\tilde{g}' B_\mu + g'' X_\mu) I. \tag{3.1}$$

where, $W_\mu \equiv \frac{\tau_i}{2} W_\mu^i$, $B_{\mu\nu}$, X_μ are SU(2)$_L$, U(1)$_Y$ and U(1)$_{Z'}$ gauge fields respectively. The reason to use X instead of Z' to label the U(1)$_{Z'}$ gauge field is due to the fact that there exists mixing among neutral gauge bosons. We denote Z' as the U(1)$_{Z'}$ gauge field after diagonaliztion. In (3.1), the new term beyond original covariant derivative given in ref. [23] is proportional to the linear combination of gauge fields B_μ and X_μ with different coefficients $\tilde{g}'$ and g''. Different choice of these coefficients will results in different Z' interactions and typical Z' dynamics from non-traditional Stueckelberg mechanism usually take $\tilde{g}' = 0$. Later, we will discuss this issue in more detail.

The full bosonic part Lagrangian up to order of p^4 is

$$\mathcal{L}_{Stueck-\text{SU(2)}_L \otimes \text{U(1)}_Y \otimes \text{U(1)}_{Z'} \to \text{U(1)}_{em}} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_4 , \tag{3.2}$$

with p^0 and p^2 order Lagrangian $\mathcal{L}_0$ and $\mathcal{L}_2$ being

$$\mathcal{L}_0 = -V(h) , \tag{3.3}$$

$$\mathcal{L}_2 = \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{4}f^2 \text{tr}\left[\hat{V}_\mu \hat{V}^\mu\right] + \frac{1}{4}\beta_1 f^2 \text{tr}\left[T\hat{V}_\mu\right] \text{tr}\left[T\hat{V}^\mu\right] + \frac{1}{4}\beta_2 f^2 \text{tr}\left[\hat{V}_\mu\right] \text{tr}\left[T\hat{V}^\mu\right]$$
$$+ \frac{1}{4}\beta_3 f^2 \text{tr}\left[\hat{V}_\mu\right] \text{tr}\left[\hat{V}^\mu\right] + \beta_4 f(\partial^\mu h) \text{tr}\left[\hat{V}_\mu\right] , \tag{3.4}$$

where $T \equiv \hat{U}\tau_3 \hat{U}^\dagger$ and $\hat{V}_\mu \equiv (\hat{D}_\mu \hat{U})\hat{U}^\dagger$. Here we treat higgs field h as p^0 order. All coefficients $f, \beta_1, \beta_2, \beta_3, \beta_4$ are functions of higgs field h. p^4 order Lagrangian $\mathcal{L}_4$ can be decomposed into four parts

$$\mathcal{L}_4 = \mathcal{L}_K + \mathcal{L}_B + \mathcal{L}_H + \mathcal{L}_A \tag{3.5}$$

in which kinetic part $\mathcal{L}_K$ is

$$\mathcal{L}_K = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}\text{tr}[W_{\mu\nu}W^{\mu\nu}] - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} . \tag{3.6}$$

Bosonic part without differential of higgs field $\mathcal{L}_B$ is

$$
\begin{aligned}
\mathcal{L}_B = {} & \frac{1}{2}\alpha_1 gg' B_{\mu\nu}\mathrm{tr}[TW^{\mu\nu}] + \frac{i}{2}\alpha_2 g' B_{\mu\nu}\mathrm{tr}\left[T\left[\hat{V}^\mu,\hat{V}^\nu\right]\right] + i\alpha_3 g\mathrm{tr}\left[W^{\mu\nu}\left[\hat{V}^\mu,\hat{V}^\nu\right]\right] \\
& + \alpha_4\mathrm{tr}\left[\hat{V}_\mu\hat{V}_\nu\right]\mathrm{tr}\left[\hat{V}^\mu\hat{V}^\nu\right] + \alpha_5\mathrm{tr}\left[\hat{V}_\mu\hat{V}^\mu\right]\mathrm{tr}\left[\hat{V}^\nu\hat{V}_\nu\right] + \alpha_6\mathrm{tr}\left[\hat{V}_\mu\hat{V}_\nu\right]\mathrm{tr}\left[T\hat{V}^\mu\right]\mathrm{tr}\left[T\hat{V}^\nu\right] \\
& + \alpha_7\mathrm{tr}\left[\hat{V}_\mu\hat{V}^\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right]\mathrm{tr}\left[T\hat{V}^\nu\right] + \frac{1}{4}\alpha_8 g^2\mathrm{tr}[TW_{\mu\nu}]\mathrm{tr}[TW^{\mu\nu}] + \frac{i}{2}\alpha_9 g\mathrm{tr}[TW^{\mu\nu}]\mathrm{tr}\left[T\left[\hat{V}_\mu,\hat{V}_\nu\right]\right] \\
& + \frac{1}{2}\alpha_{10}\mathrm{tr}\left[T\hat{V}^\mu\right]\mathrm{tr}\left[T\hat{V}^\nu\right]\mathrm{tr}\left[T\hat{V}_\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right] + \alpha_{11}g\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}\left[T\hat{V}_\mu\right]\mathrm{tr}\left[\hat{V}_\nu W_{\rho\lambda}\right] \\
& + \alpha_{12}g\mathrm{tr}\left[T\hat{V}^\mu\right]\mathrm{tr}\left[\hat{V}^\nu W_{\mu\nu}\right] + \alpha_{13}gg'\epsilon^{\mu\nu\rho\lambda}B_{\mu\nu}\mathrm{tr}[TW_{\rho\lambda}] + \alpha_{14}g^2\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}[TW_{\mu\nu}]\mathrm{tr}[TW_{\rho\lambda}] \\
& + \alpha_{15}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[T\hat{V}^\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right]\mathrm{tr}\left[T\hat{V}^\nu\right] + \alpha_{16}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[T\hat{V}^\mu\right]\mathrm{tr}\left[\hat{V}_\nu\hat{V}^\nu\right] \\
& + \alpha_{17}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right]\mathrm{tr}\left[\hat{V}^\mu\hat{V}^\nu\right] + \alpha_{18}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[\hat{V}_\nu\right]\mathrm{tr}\left[T\hat{V}^\mu\right]\mathrm{tr}\left[T\hat{V}^\nu\right] + \alpha_{19}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[\hat{V}_\nu\right]\mathrm{tr}\left[\hat{V}^\mu\hat{V}^\nu\right] \\
& + \alpha_{20}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[\hat{V}^\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right]\mathrm{tr}\left[T\hat{V}^\nu\right] + \alpha_{21}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[\hat{V}^\mu\right]\mathrm{tr}\left[\hat{V}_\nu\hat{V}^\nu\right] \\
& + \alpha_{22}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[\hat{V}^\mu\right]\mathrm{tr}\left[\hat{V}_\nu\right]\mathrm{tr}\left[T\hat{V}^\nu\right] + \alpha_{23}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[\hat{V}_\nu\right]\mathrm{tr}\left[\hat{V}^\mu\right]\mathrm{tr}\left[\hat{V}^\nu\right] + gg''\alpha_{24}X_{\mu\nu}\mathrm{tr}[TW^{\mu\nu}] \\
& + g'g''\alpha_{25}B_{\mu\nu}X^{\mu\nu} + \alpha_{26}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right]\mathrm{tr}\left[T\left[\hat{V}_\rho,\hat{V}_\lambda\right]\right] + ig'\alpha_{27}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right]B_{\rho\lambda} \\
& + ig\alpha_{28}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right]\mathrm{tr}[TW_{\rho\lambda}] + g\alpha_{29}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[\hat{V}_\nu W_{\rho\lambda}\right] \\
& + ig''\alpha_{30}\epsilon^{\mu\nu\rho\lambda}X_{\mu\nu}\mathrm{tr}\left[T\left[\hat{V}_\rho,\hat{V}_\lambda\right]\right] + ig''\alpha_{31}X_{\mu\nu}\mathrm{tr}\left[T\left[\hat{V}^\mu,\hat{V}^\nu\right]\right] + g''\alpha_{32}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right]X_{\rho\lambda} \\
& + \alpha_{33}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right]\mathrm{tr}\left[T\left[\hat{V}^\mu,\hat{V}^\nu\right]\right] + g'g''\alpha_{34}\epsilon^{\mu\nu\rho\lambda}B_{\mu\nu}X_{\rho\lambda} + gg''\alpha_{35}\epsilon^{\mu\nu\rho\lambda}X_{\mu\nu}\mathrm{tr}[TW_{\rho\lambda}] \\
& + ig'\alpha_{36}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right]B^{\mu\nu} + ig\alpha_{37}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right]\mathrm{tr}[TW^{\mu\nu}] + g\alpha_{38}\mathrm{tr}\left[\hat{V}^\mu\right]\mathrm{tr}\left[\hat{V}^\nu W_{\mu\nu}\right] \\
& + g''\alpha_{39}\mathrm{tr}\left[\hat{V}_\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right]X^{\mu\nu} + ig\alpha_{40}\mathrm{tr}\left[\hat{V}^\mu\right]\mathrm{tr}\left[T\hat{V}^\nu W_{\mu\nu}\right] \, .
\end{aligned}
\tag{3.7}
$$

Among them $\alpha_{12} \sim \alpha_{14}$, α_{30}, $\alpha_{33} \sim \alpha_{40}$ are CP-violation terms. Bosonic part with differential of higgs field $\mathcal{L}_H$ is

$$
\begin{aligned}
\mathcal{L}_H = {} & (\partial_\mu h)\Big\{\alpha_{H,1}\mathrm{tr}\left[T\hat{V}^\mu\right]\mathrm{tr}\left[\hat{V}_\nu\hat{V}^\nu\right] + \alpha_{H,2}\mathrm{tr}\left[T\hat{V}_\nu\right]\mathrm{tr}\left[\hat{V}^\mu\hat{V}^\nu\right] + \alpha_{H,3}\mathrm{tr}\left[T\hat{V}_\nu\right]\mathrm{tr}\left[T\left[\hat{V}^\mu,\hat{V}^\nu\right]\right] \\
& + \alpha_{H,4}\mathrm{tr}\left[T\hat{V}^\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right]\mathrm{tr}\left[T\hat{V}^\nu\right] + ig\alpha_{H,5}\mathrm{tr}\left[T\hat{V}_\nu\right]\mathrm{tr}[TW^{\mu\nu}] + g'\alpha_{H,6}\mathrm{tr}\left[T\hat{V}_\nu\right]B^{\mu\nu} \\
& + ig\alpha_{H,7}\mathrm{tr}\left[T\hat{V}_\nu W^{\mu\nu}\right] + g\alpha_{H,8}\mathrm{tr}\left[\hat{V}_\nu W^{\mu\nu}\right] + \alpha_{H,9}\mathrm{tr}\left[\hat{V}^\mu\right]\mathrm{tr}\left[\hat{V}_\nu\hat{V}^\nu\right] + \alpha_{H,10}\mathrm{tr}\left[\hat{V}_\nu\right]\mathrm{tr}\left[\hat{V}^\mu\hat{V}^\nu\right] \\
& + \alpha_{H,11}\mathrm{tr}\left[\hat{V}_\nu\right]\mathrm{tr}\left[T\left[\hat{V}^\mu,\hat{V}^\nu\right]\right] + \alpha_{H,12}\mathrm{tr}\left[\hat{V}^\mu\right]\mathrm{tr}\left[T\hat{V}_\nu\right]\mathrm{tr}\left[T\hat{V}^\nu\right] + \alpha_{H,13}\mathrm{tr}\left[\hat{V}^\mu\right]\mathrm{tr}\left[\hat{V}_\nu\right]\mathrm{tr}\left[T\hat{V}^\nu\right] \\
& + ig\alpha_{H,14}\mathrm{tr}\left[\hat{V}_\nu\right]\mathrm{tr}[TW^{\mu\nu}] + g'\alpha_{H,15}\mathrm{tr}\left[\hat{V}_\nu\right]B^{\mu\nu}\Big\} + (\partial_\mu h)(\partial_\nu h)\Big\{\alpha_{H,16}\mathrm{tr}\left[T\hat{V}^\mu\right]\mathrm{tr}\left[T\hat{V}^\nu\right] \\
& + \alpha_{H,17}\mathrm{tr}\left[\hat{V}^\mu\hat{V}^\nu\right] + \alpha_{H,18}\mathrm{tr}\left[\hat{V}^\mu\right]\mathrm{tr}\left[T\hat{V}^\nu\right] + \alpha_{H,19}\mathrm{tr}\left[\hat{V}^\mu\right]\mathrm{tr}\left[\hat{V}^\nu\right]\Big\} + (\partial_\mu h)(\partial^\mu h) \\
& \times \Big\{\alpha_{H,20}\mathrm{tr}\left[T\hat{V}_\nu\right]\mathrm{tr}\left[T\hat{V}^\nu\right] + \alpha_{H,21}\mathrm{tr}\left[\hat{V}_\nu\hat{V}^\nu\right] + \alpha_{H,22}\mathrm{tr}\left[\hat{V}_\nu\right]\mathrm{tr}\left[T\hat{V}^\nu\right] + \alpha_{H,23}\mathrm{tr}\left[\hat{V}_\nu\right]\mathrm{tr}\left[\hat{V}^\nu\right]\Big\} \\
& + (\partial_\mu h)(\partial^\mu h)(\partial_\nu h)\Big\{\alpha_{H,24}\mathrm{tr}\left[T\hat{V}^\nu\right] + \alpha_{H,25}\mathrm{tr}\left[\hat{V}^\nu\right]\Big\} + \alpha_{H,26}[(\partial_\mu h)(\partial^\mu h)]^2 \, .
\end{aligned}
\tag{3.8}
$$

Anomaly part $\mathcal{L}_A$ is

$$
\mathcal{L}_A = \alpha_{42}g^2\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}\left[W_{\mu\nu}W_{\rho\lambda}\right] + \alpha_{43}g'^2\epsilon^{\mu\nu\rho\lambda}B_{\mu\nu}B_{\rho\lambda} + g''^2\alpha_{44}\epsilon^{\mu\nu\rho\lambda}X_{\mu\nu}X_{\rho\lambda} \, .
\tag{3.9}
$$

Similar as p^0 and p^2 order, all α coefficients in p^4 order Lagrangian are functions of higgs field h.

Above chiral Lagrangian is the most general EWCL involve Z' and higgs fields, in terms of which we can examine details of Z' physics. In following of this section, we focus our attentions on the mixing among gauge bosons.

We take unitary gauge $\hat{U} = 1$. The gauge boson mass term $\mathcal{L}_M$ and kinetic term $\mathcal{L}_K$ become

$$\mathcal{L}_M = \frac{1}{8} f^2 g^2 \Big[W_\mu^1 W^{1,\mu} + W_\mu^2 W^{2,\mu} \Big] + \frac{1}{8}(1 - 2\beta_1) f^2 (g W_\mu^3 - g' B_\mu)(g W^{3,\mu} - g' B^\mu) \quad (3.10)$$

$$+ \frac{1}{2}(1 - 2\beta_3) f^2 (g'' X_\mu + \tilde{g}' B_\mu)(g'' X^\mu + \tilde{g}' B^\mu) + \frac{1}{2} \beta_2 f^2 (g'' X_\mu + \tilde{g}' B_\mu)(g W^{3,\mu} - g' B^\mu) \, ,$$

$$\mathcal{L}_K = -\frac{1}{4} B_{\mu\nu} B_{\mu\nu} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{4}(\partial_\mu W_\nu^1 - \partial_\nu W_\mu^1)^2 - \frac{1}{4}(\partial_\mu W_\nu^2 - \partial_\nu W_\mu^2)^2$$

$$-\frac{1}{4}(1 - \alpha_8 g^2)(\partial_\mu W_\nu^3 - \partial_\nu W_\mu^3)^2 + \frac{1}{2} \alpha_1 g g' B_{\mu\nu}(\partial_\mu W_\nu^3 - \partial_\nu W_\mu^3)$$

$$+ g g'' \alpha_{24} X^{\mu\nu}(\partial_\mu W_\nu^3 - \partial_\nu W_\mu^3) + g' g'' \alpha_{25} B_{\mu\nu} X^{\mu\nu} \, , \quad (3.11)$$

in which the charged gauge bosons W_μ^1 and W_μ^2 are automatically diagonalized. This is due to the fact that there is no other charged vector and scalar particles to mix with. To generate mixing for charged gauge bosons, we need to add in theory new charged gauge bosons, such as $W_\mu'^{,1}$ and $W_\mu'^{,2}$ which was already discussed in ref. [27] or new charged Higgs bosons. For the remaining neutral gauge bosons W^3, B, X, our Lagrangian includes most general mixing among them. We can choose special parameters to recover the various scenarios discussed in literature. For example,

- Taking $f g'' = M_1$, $f \tilde{g}' = M_2$ and $\alpha_i = \beta_j = 0$, (3.10) and (3.11) come back to Stueckelberg Lagrangian given in ref. [4] which depends on coefficients M_1 and M_2 .

- Taking $\frac{\beta_2 \sqrt{g^2 + g'^2}}{2(1 - \beta_3) g''} = x$, $-2g''(g\alpha_{24} - g'\alpha_{25}) = y$, $-2g''(g\alpha_{24} + g'\alpha_{25}) = w$, $(1 - 2\beta_3)g''^2 f^2 = m_X^2$ and $\beta_1 = \alpha_i = 0$ $(i \neq 24, 25)$, (3.10) and (3.11) come back to effective Lagrangian given in ref. [32] which depends on coefficients x, y, w, m_X^2 and includes a more simplified case discussed in an earlier ref. [33].

- Taking $-2g g'' \alpha_{25} = \sin\chi$, and $\beta_1 = \beta_3 = \alpha_i = 0$ $(i \neq 25)$. (3.10) and (3.11) come back to effective Lagrangian for E_6 model given in ref. [34] which depends on a mixing angle χ .

What we need to do next is to diagonalize these mass and kinetic terms. We first try to cancel term $(g'' X_\mu + \tilde{g}' B_\mu)(g W_\mu^3 - g' B_\mu)$ in $\mathcal{L}_M$ by mixing $g'' X + \tilde{g}' B$ with $g W_\mu^3 - g' B_\mu$

$$g'' X_\mu + \tilde{g}' B_\mu = \cos\alpha_{Z'} \left(g'' \bar{X}_\mu + \tilde{g}' \bar{B}_\mu \right) + \sin\alpha_{Z'} \left(g \bar{W}_\mu^3 - g' \bar{B}_\mu \right)$$

$$g W_\mu^3 - g' B_\mu = -\sin\alpha_{Z'} \left(g'' \bar{X}_\mu + \tilde{g}' \bar{B}_\mu \right) + \cos\alpha_{Z'} \left(g \bar{W}_\mu^3 - g' \bar{B}_\mu \right) \, , \quad (3.12)$$

where

$$\tan\alpha_{Z'} = \frac{3 + 2\beta_1 - 8\beta_3 - \sqrt{(3 + 2\beta_1 - 8\beta_3)^2 + 16\beta_2^2}}{4\beta_2}. \quad (3.13)$$

$\mathcal{L}_M$ then reads as

$$\mathcal{L}_M = \frac{1}{8}f^2 g^2 \left[W_\mu^1 W^{1,\mu} + W_\mu^2 W^{2,\mu}\right] + \frac{1}{2}A_2^2 f^2 (g'' \bar{X}_\mu + \tilde{g}' \bar{B}_\mu)^2 + \frac{1}{2}A_1^2 f^2 (g\bar{W}_\mu^3 - g'\bar{B}_\mu)^2, \quad (3.14)$$

with

$$A_1^2 = \frac{1}{4}(1 - 2\beta_1)c_\alpha^2 + \beta_2 s_\alpha c_\alpha + (1 - 2\beta_3)s_\alpha^2 \quad (3.15)$$

$$A_2^2 = \frac{1}{4}(1 - 2\beta_1)s_\alpha^2 - \beta_2 s_\alpha c_\alpha + (1 - 2\beta_3)c_\alpha^2 \, , \quad (3.16)$$

where, $c_\alpha \equiv \cos\alpha_{Z'}$ and $s_\alpha \equiv \sin\alpha_{Z'}$.

The kinetic term for neutral gauge boson can be written as

$$\mathcal{L}_{\mathrm{K,neutral}} = (W_{\mu\nu}^3, B_{\mu\nu}, X_{\mu\nu}) \begin{pmatrix} -\frac{1}{4}(1 - \alpha_8 g^2) & \frac{1}{4}\alpha_1 gg' & \frac{1}{2}gg''\alpha_{24} \\ \frac{1}{4}\alpha_1 gg' & -\frac{1}{4} & \frac{1}{2}g'g''\alpha_{25} \\ \frac{1}{2}gg''\alpha_{24} & \frac{1}{2}g'g''\alpha_{25} & -\frac{1}{4} \end{pmatrix} \begin{pmatrix} W^{3\mu\nu} \\ B^{\mu\nu} \\ X^{\mu\nu} \end{pmatrix} \, . \quad (3.17)$$

Decompose gW_μ^3 as $(gW_\mu^3 - g'B_\mu)/2 + (gW_\mu^3 + g'B_\mu)/2$, $g'B_\mu$ as $-(gW_\mu^3 - g'B_\mu)/2 + (gW_\mu^3 + g'B_\mu)/2$ and $g''X_\mu$ as $g''X_\mu + \tilde{g}'B_\mu + (gW_\mu^3 - g'B_\mu)\tilde{g}'/2g' - (gW_\mu^3 + g'B_\mu)\tilde{g}'/2g'$. With help of (3.12), we find

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \\ X_\mu \end{pmatrix} = \begin{pmatrix} \frac{1}{2g}c_\alpha & \frac{1}{2g} & -\frac{1}{2g}s_\alpha \\ -\frac{1}{2g'}c_\alpha & \frac{1}{2g'} & \frac{1}{2g'}s_\alpha \\ \frac{1}{g''}\left(s_\alpha + \frac{\tilde{g}'}{2g'}c_\alpha\right) & -\frac{\tilde{g}'}{2g''g'} & \frac{1}{g''}\left(c_\alpha - \frac{\tilde{g}'}{2g'}s_\alpha\right) \end{pmatrix} \begin{pmatrix} g\bar{W}_\mu^3 - g'\bar{B}_\mu \\ gW_\mu^3 + g'B_\mu \\ g''\bar{X}_\mu + \tilde{g}'\bar{B}_\mu \end{pmatrix} \, .$$

Further take following transformation which keeps neutral gauge boson mass terms to be diagonal and rotates neutral gauge boson to the basis of Z_μ, photon A_μ and Z'_μ

$$\begin{pmatrix} g\bar{W}_\mu^3 - g'\bar{B}_\mu \\ gW_\mu^3 + g'B_\mu \\ g''\bar{X}_\mu + \tilde{g}'\bar{B}_\mu \end{pmatrix} = \begin{pmatrix} \frac{\cos\beta_{Z'}}{A_1} & 0 & \frac{\sin\beta_{Z'}}{A_1} \\ ga & gb & gc \\ -\frac{\sin\beta_{Z'}}{A_2} & 0 & \frac{\cos\beta_{Z'}}{A_2} \end{pmatrix} \begin{pmatrix} \frac{M_Z}{f}Z_\mu \\ A_\mu \\ \frac{M_{Z'}}{f}Z'_\mu \end{pmatrix} \, . \quad (3.18)$$

Then the mass term involving neutral gauge bosons can be written as

$$\mathcal{L}_{\mathrm{M,neutral}} = \frac{1}{2}A_1^2 f^2 (g\bar{W}_\mu^3 - g'\bar{B}_\mu) + \frac{1}{2}A_1^2 f^2 \bar{X}_\mu^2 = \frac{1}{2}M_Z^2 Z_\mu^2 + \frac{1}{2}M_{Z'}^2 Z_\mu'^2 \, , \quad (3.19)$$

with massless photon. Remaining six parameters are Z mass M_Z, Z' mass $M_{Z'}$, mixing angle $\beta_{Z'}$ and coefficients a, b, c, which will be determined later. Now total rotation matrix becomes

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \\ X_\mu \end{pmatrix} = U \begin{pmatrix} Z_\mu \\ A_\mu \\ Z'_\mu \end{pmatrix} \quad (3.20)$$

with

$$U \equiv \begin{pmatrix} \frac{1}{2g}c_\alpha & \frac{1}{2g} & -\frac{1}{2g}s_\alpha \\ -\frac{1}{2g'}c_\alpha & \frac{1}{2g'} & \frac{1}{2g'}s_\alpha \\ \frac{1}{g''}\left(s_\alpha + \frac{\tilde{g}'}{2g'}c_\alpha\right) & -\frac{\tilde{g}'}{2g''g'} & \frac{1}{g''}\left(c_\alpha - \frac{\tilde{g}'}{2g'}s_\alpha\right) \end{pmatrix} \begin{pmatrix} \frac{c_\beta}{A_1} & 0 & \frac{s_\beta}{A_1} \\ ga & gb & gc \\ -\frac{s_\beta}{A_2} & 0 & \frac{c_\beta}{A_2} \end{pmatrix} \begin{pmatrix} \frac{M_Z}{f} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{M_{Z'}}{f} \end{pmatrix} \, , \quad (3.21)$$

where, $s_\beta = \sin \beta_{Z'}$ and $c_\beta = \cos \beta_{Z'}$. With above rotation, kinetic term for neutral gauge boson (3.17) can be further written as

$$\mathcal{L}_{\text{K,neural}} = (Z_{\mu\nu}, A_{\mu\nu}, Z'_{\mu\nu})\mathbf{K}\begin{pmatrix} Z^{\mu\nu} \\ A^{\mu\nu} \\ Z'^{,\mu\nu} \end{pmatrix} \quad \mathbf{K} \equiv U^T \begin{pmatrix} -\frac{1}{4}(1 - \alpha_8 g^2) & \frac{1}{4}\alpha_1 g g' & \frac{1}{2}g g'' \alpha_{24} \\ \frac{1}{4}\alpha_1 g g' & -\frac{1}{4} & \frac{1}{2}g' g'' \alpha_{25} \\ \frac{1}{2}g g'' \alpha_{24} & \frac{1}{2}g' g'' \alpha_{25} & -\frac{1}{4} \end{pmatrix} U.$$

(3.22)

In which $\mathbf{K}$ is three by three symmetric matrix. Denote its matrix elements as $\mathbf{K}_{ij}$. Notice that $\mathbf{K}_{11} \propto M_Z^2/f^2$ and $\mathbf{K}_{33} \propto M_{Z'}^2/f^2$, then normalization of Z and Z' kinetic terms,

$$\mathbf{K}_{11} = -\frac{1}{4} \qquad\qquad \mathbf{K}_{33} = -\frac{1}{4}\,,$$

(3.23)

is necessary to interpret M_Z and $M_{Z'}$ introduced in (3.19) as the correct definition of Z and Z' masses. Above normalization condition also fix values of M_Z and $M_{Z'}$. Remaining normalization of photon kinetic term demands $\mathbf{K}_{22} = -1/4$ and diagonalization of kenetic terms requires $\mathbf{K}_{12} = \mathbf{K}_{13} = \mathbf{K}_{23} = 0$. These four constraint conditions further fix remaining four parameters $\beta_{Z'}, a, b, c$. Detailed computation shows that first $\mathbf{K}_{22} = -\frac{1}{4}$ fix parameter b,

$$b^2 = \frac{4g'^2 g''^2}{(g^2 + g'^2)g''^2 + g^2 \tilde{g}'^2 - g^2 g'^2 g''^2 (2\alpha_1 + \alpha_8) + 4g^2 g' g''^2 \tilde{g}'(\alpha_{24} + \alpha_{25})}\,.$$

while $\mathbf{K}_{12} = 0$ fix parameter a,

$$a = \frac{1}{g A_1 A_2 \left[g'^2 g''^2 - g^2 g'^2 g''^2 (2\alpha_1 + \alpha_8) + g^2 g''^2 - 4g^2 g' g''^2 \tilde{g}'(\alpha_{24} + \alpha_{25}) + g^2 \tilde{g}'^2\right]}$$
$$\times \Big\{ \left[g^2 g''^2 + g^2 \tilde{g}'^2 - g'^2 g''^2 + g^2 g'^2 g''^2 \alpha_8 + 4g^2 g' g''^2 \tilde{g}' \alpha_{25}\right](s_\alpha s_\beta A_1 + c_\alpha c_\beta A_2)$$
$$+ \left[2g^2 g' \tilde{g}' + 4g^2 g'^2 g''^2 (\alpha_{24} + \alpha_{25})\right](-c_\alpha s_\beta A_1 + s_\alpha c_\beta A_2)\Big\}\,.$$

$\mathbf{K}_{23} = 0$ fix parameter c,

$$c = \frac{1}{g A_1 A_2 \left[g'^2 g''^2 - g^2 g'^2 g''^2 (2\alpha_1 + \alpha_8) + g^2 g''^2 - 4g^2 g' g''^2 \tilde{g}'(\alpha_{24} + \alpha_{25}) + g^2 \tilde{g}'^2\right]}$$
$$\times \Big\{ \left[g^2 g''^2 + g^2 \tilde{g}'^2 - g'^2 g''^2 + g^2 g'^2 g''^2 \alpha_8 + 4g^2 g' g''^2 \tilde{g}' \alpha_{25}\right](-s_\alpha c_\beta A_1 + c_\alpha s_\beta A_2)$$
$$+ \left[2g^2 g' \tilde{g}' + 4g^2 g'^2 g''^2 (\alpha_{24} + \alpha_{25})\right](c_\alpha c_\beta A_1 + s_\alpha s_\beta A_2)\Big\}\,.$$

Finally $\mathbf{K}_{1,3} = 0$ gives constraint

$$0 = G_0 \left(1 - 2c_\beta^2\right) + G_2 s_\beta c_\beta\,,$$

(3.24)

with G_0 and G_1 given in (A.1). Eq. (3.24) yielding $\tan \beta$

$$\tan \beta = \frac{-G_2 + \sqrt{G_2^2 + 4G_0^2}}{2G_0}\,.$$

(3.25)

Since the precision of our computation is only accurate up order of p^4, while β_i, $i = 1, 2, 3, 4$ represent p^2 order operators and α_i represent p^4 order operators. Therefore we can expand our result in powers of β_i and α_i and to our p^4 order precision, we only need to keep terms at most quadratic in β_i and linear in α_i. Detailed computation gives $\tan\alpha_{Z'} = -2\beta_2/(3-2\beta_1+8\beta_3)$, $A_1 = (1-\beta_1)/2 - \beta_2^2/3 - \beta_1^2/4$ and $A_2 = 1 - \beta_3 + \beta_2^2/6 - \beta_3^2/2$. The rotation matrix U is given in (A.2). In literature, most models [12, 28–30] treat extra X gauge boson by only mixing it with W^3 boson in mass term. This corresponds to $\alpha_1 = \alpha_8 = \alpha_{24} = \alpha_{25} = \beta_1 = \beta_3 = \tilde{g}' = 0$ in our EEWCL. The result mixing angle become

$$\tan\xi = \frac{4g''\sqrt{g^2+g'^2}}{g^2+g'^2-4g''^2}\beta_2 \ . \tag{3.26}$$

Usually constraints on ξ are highly model-dependent [31], the typical value of which is at order of 10^{-3}. In our general case, X boson can mix not only with W^3, but also with B, *"There are no quantum numbers which forbid a mixing of neutral gauge bosons", Leike1999.* In Leike's review article [10], general mixing among X_μ, W_μ^3 and B_μ in mass terms is parameterized. Further mixing can happen not only in mass terms, but also in kinetic terms. Authors in ref. [32, 35] studied a case that in kinetic terms, there are mixing among X_μ, W_μ^3 and B_μ. In our formulation, we use three by three U matrix to parameterize the most general mixing among X_μ, W_μ^3 and B_μ happened both in mass terms and kinetic terms. The small values for mixing among X with W^3 and B require smallness in values for $U_{1,3}, U_{2,3}, U_{3,1}, U_{3,2}$, which from (A.2) leads to the requirements

$$4g''^2 \neq g^2 + g'^2 \qquad \tilde{g}' \ll 1 \qquad g''\beta_2 \ll 1 \qquad g''\alpha_{24} \ll 1 \qquad g''\alpha_{25} \ll 1 \ . \tag{3.27}$$

Another sector which heavily depends on W^3, B and X mixing is the neutral current. The corresponding Lagrangian is $gW_\mu^3 J^{3,\mu} + g'B_\mu J_Y^\mu + g''X_\mu J_X^\mu$, in which except conventional weak isospin third component current $J^{3,\mu}$ and hypercharge current J_Y^μ, we now have extra hidden current J_X^μ couple to X_μ boson. In terms of physical gauge boson Z, A, Z', the Lagrangian becomes $eJ_{\rm em}^\mu A_\mu + g_Z J_Z^\mu Z_\mu + g''J_{Z'}^\mu Z'_\mu$. With help of (3.21), we can read out

$$
\begin{aligned}
eJ_{\rm em}^\mu &= gU_{1,2}J^{3,\mu} + g'U_{2,2}J_Y^\mu + g''U_{3,2}J_X^\mu \\
g_Z J_Z^\mu &= gU_{1,1}J^{3,\mu} + g'U_{2,1}J_Y^\mu + g''U_{3,1}J_X^\mu \\
g''J_{Z'}^\mu &= gU_{1,3}J^{3,\mu} + g'U_{2,3}J_Y^\mu + g''U_{3,3}J_X^\mu \ .
\end{aligned}
\tag{3.28}
$$

For which, we find

- When $\tilde{g}' \neq 0$, due to fact $U_{3,2} \neq 0$ given in (A.2), photon will couple to hidden neutral current J_X^μ. This situation was discussed in ref. [2].

- Small mixing among X with W^3 and B achieved by (3.27) will imply that hidden neutral current J_X^μ decouples from Z boson and photon approximately; $J^{3,\mu}$ and J_Y^μ also decouple from Z' boson approximately.

- J_X^μ mainly couples to Z' and the coupling is g'' which will see later that is proportional to $M_{Z'}/f$.

We now display the last three parameters accurate up order of p^4 and linear order of $\tilde{g}'$

$$
\begin{aligned}
\beta_{Z'} = {} & \frac{1}{\Delta_g}\left\{2g'\tilde{g}' - \frac{1}{3}\left(3g_Z^2 - \Delta_g\right)\beta_2 - 4g''^2\left(g^2\alpha_{24} - g'^2\alpha_{25}\right)\right\} - \frac{2}{9\Delta_g{}^2}\left\{-9g'\left(2g_Z^2 - \Delta_g\right)\tilde{g}'(\beta_1 - \beta_3)\right. \\
& \left. + 2\left(g_Z^2 - g''^2\right)\beta_2\left[\left(g_Z^2 + 20g''^2\right)\beta_1 + \left(5g_Z^2 - 44g''^2\right)\beta_3\right]\right\} - \frac{2g^2\tilde{g}'g'\left(\Delta_g - 2g'^2\right)\alpha_1}{\Delta_g{}^2} \\
& - \frac{2g'\tilde{g}'g^4\alpha_8}{\Delta_g{}^2} - \frac{2g'\tilde{g}'\left(g_Z^4 + 24g_Z^2 g''^2 + 16g''^4\right)\beta_3\beta_1}{\Delta_g{}^3} + \frac{g'\left(3g_Z^4 + 24g_Z^2 g''^2 - 16g''^4\right)\tilde{g}'\beta_1{}^2}{\Delta_g{}^3} \\
& + \frac{1}{3}\frac{\tilde{g}'\left(29g_Z^4 - 16\,g''^4 - 136g_Z^2 g''^2\right)g'\beta_2{}^2}{\Delta_g{}^3} - \frac{g'\left(-24g_Z^2 g''^2 - 48\,g''^4 + g_Z^4\right)\tilde{g}'\beta_3{}^2}{\Delta_g{}^3}
\end{aligned}
$$

$$
\begin{aligned}
\frac{M_Z^2}{f^2} = {} & \frac{1}{4}\left[g_Z^2\left(1 - 2\beta_1\right) - 2g^2 g'^2\alpha_1 + g^4\alpha_8\right] - \frac{g_Z^2\left(g'\tilde{g}'\beta_2 - g''^2\beta_2^2\right)}{\Delta_g} \\
& - \frac{4\tilde{g}'g'g''^2\left(g^2\alpha_{24} - g'^2\alpha_{25}\right)}{\Delta_g} - \frac{8g_Z^2 g''^2 g'\tilde{g}'}{\Delta_g{}^2}(\beta_2\beta_1 - \beta_2\beta_3)
\end{aligned}
$$

$$
\begin{aligned}
\frac{M_{Z'}^2}{f^2} = {} & g''^2(1 - 2\beta_3) + \frac{g''^2\left[4g'\tilde{g}'\beta_2 - g_Z^2\beta_2^2\right]}{\Delta_g} + 4\frac{g''^2\tilde{g}'g'\left[g^2\alpha_{24} - \left(g^2 - 4g''^2\right)\alpha_{25}\right]}{\Delta_g} \\
& + 8\frac{g''^2 g'\tilde{g}'g_Z^2}{\Delta_g{}^2}(\beta_2\beta_1 - \beta_2\beta_3)\,,
\end{aligned} \tag{3.29}
$$

where $g_Z = \sqrt{g^2 + g'^2}$ and $\Delta_g = g^2 + g'^2 - 4g''^2$. All β_i and α_i coefficients appear in above results must take their values with Higgs field inside the coefficients being substituted by its vacuum expectation value. Notice that the correction for Z mass from extra Z' couplings is proportional to $\tilde{g}'\beta_2$, $(g''\beta_2)^2$, $g''\alpha_{24}$ and $g''\alpha_{25}$ which are very small if we adopt (3.27). In fact, in formulae for M_Z and $M_{Z'}$, if we ignore these small mixing and further neglect contribution from $\beta_1, \alpha_1, \alpha_8$ which roughly are related to phenomenological parameters T, S, U [23], we find $M_{Z'}^2/M_Z \sim 2g''^2(1 - 2\beta_3)/e^2$. This implies that even for small mixing for neutral gauge bosons with Z' we still have two independent parameters g'' and β_3 to tune its value.

We finish discussion on mixing among neutral gauge bosons by checking our computation results. With constraints (3.27), X mixing with W^3 and B controlled by parameter $\tilde{g}'$, $g''\beta_2$, $g''\alpha_{24}$ and $g''\alpha_{25}$ become very small. Ignoring contributions from these parameters, X will not mix with W^3 and B any more and the left mixing between W^3 and B then goes back to its value given be standard EWCL [23]. If we further demand $\tilde{g}' = \beta_1 = \beta_2 = \beta_3 = \alpha_1 = \alpha_8 = \alpha_{24} = \alpha_{25} = 0$, we recover results of tree diagram SM which include $G_0 = 0$, $A_1 = 1/2$, $A_2 = 1$ and the matrix U becomes

$$
\begin{pmatrix}
\cos\theta_W & \sin\theta_W & 0 \\
-\sin\theta_W & \cos\theta_W & 0 \\
0 & 0 & 1
\end{pmatrix}, \tag{3.30}
$$

with $\tan\theta_W = g'/g$. The six parameters at this order of approximation are $\alpha_{Z'} = \beta_{Z'} = c = 0$, $a = 2\frac{g^2 - g'^2}{g(g^2 + g'^2)}$, $b = \frac{2g'}{\sqrt{g^2 + g'^2}}$, $M_Z = \frac{f}{2}\sqrt{g^2 + g'^2}$ and $M_{Z'} = g''f$.

4. Anomalous couplings among gauge fields

In this section, we discuss effective gauge boson self interactions which include triple and quartic coupling terms and these terms which not only include SM electroweak gauge fields $W^\pm, A, Z$, but also involve Z' field. The part without Z' field can be parameterized by coefficients in original EWCL and parametrization for quadratic and triple couplings were already given in ref. [23]. The quartic couplings can be worked out as follows,

$$\mathcal{L}_{\text{QGV}} = g_{++--}W_\mu^+ W^{+\mu}W_\nu^- W^{-\nu} - g_{+-+-}(W_\mu^+ W^{-\mu})^2 + g_{Z^4}(Z_\mu Z^\mu)^2 + g_{+Z-Z}W_\mu^+ Z^\mu W_\nu^- Z^\nu$$
$$- g_{+-ZZ}W_\mu^+ W^{-\mu}Z_\nu Z^\nu - g_{+-ZA}W_\mu^+ W^{-\mu}Z_\nu A^\nu - ig_{\epsilon+-ZA}\epsilon^{\mu\nu\rho\lambda}Z_\mu A_\nu W_\rho^+ W_\lambda^-$$
$$+ [g_{+Z-A\oplus+A-Z}(W_\mu^+ W_\nu^- + W_\nu^+ W_\mu^-) + ig_{+Z-A\ominus+A-Z}(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-)]Z^\mu A^\nu$$
$$- e^{*2}(A_\mu A^\mu W^{+\nu}W_\nu^- - W_\mu^- W_\nu^+ A^\mu A^\nu) ,$$

with nine anomalous quartic couplings determined by

$$g_{++--} = \frac{e^{*2}}{8\sin^2\theta_W}|_Z \left[4 + \frac{2\alpha_1 e^2}{c^2 - s^2} + \alpha_8 e^2 \left(-\frac{1}{s^2} + \frac{c^2}{c^2 - s^2}\right)\right.$$
$$\left. + \frac{2\beta_1 c^2}{c^2 - s^2} + \left(\alpha_3 + \frac{1}{2}\alpha_4 - \frac{1}{2}\alpha_8 + \alpha_9\right)\frac{8e^2}{s^2}\right]$$

$$g_{+-+-} = -\frac{e^{*2}}{8\sin^2\theta_W}|_Z \left[4 + \frac{2\alpha_1 e^2}{c^2 - s^2} + \alpha_8 e^2 \left(-\frac{1}{s^2} + \frac{c^2}{c^2 - s^2}\right)\right.$$
$$\left. + \frac{2\beta_1 c^2}{c^2 - s^2} - \left(-\alpha_3 + \frac{1}{2}\alpha_4 + \alpha_5 + \frac{1}{2}\alpha_8 - \alpha_9\right)\frac{8e^2}{s^2}\right]$$

$$g_{Z^4} = e^{*4}\cot^4\theta_W|_Z \left(\frac{1}{2}\alpha_4 + \frac{1}{2}\alpha_5 + \alpha_6 + \alpha_7 + \alpha_{10}\right)\frac{1}{2c^8}$$

$$g_{+Z-Z} = e^{*2}\cot^2\theta_W|_Z \left[1 + \frac{2\beta_1}{c^2 - s^2} + \frac{2}{c^2(c^2 - s^2)}e^2\alpha_1 + \left(2\alpha_3 + \frac{\alpha_4 + \alpha_6}{c^2}\right)\frac{e^2}{s^2 c^2}\right]$$

$$g_{+-ZZ} = e^{*2}\cot^2\theta_W|_Z \left[1 + \frac{2\beta_1}{c^2 - s^2} + \frac{2}{c^2(c^2 - s^2)}e^2\alpha_1 + \left(2\alpha_3 - \frac{\alpha_5 + \alpha_7}{c^2}\right)\frac{e^2}{s^2 c^2}\right]$$

$$g_{+-ZA} = 2\left[1 + \frac{\beta_1}{c^2 - s^2} + \frac{e^2\alpha_1}{c^2(c^2 - s^2)} + \alpha_3\frac{e^2}{s^2 c^2}\right] \qquad g_{\epsilon+-ZA} = \frac{2e^2}{s^2 c^2}\alpha_{11} \qquad (4.1)$$

$$g_{+Z-A\oplus+A-Z} = e^{*2}\cot\theta_W|_Z \left[1 + \frac{\beta_1}{c^2 - s^2} + \frac{1}{c^2(c^2 - s^2)}e^2\alpha_1 + \alpha_3\frac{e^2}{s^2 c^2}\right]$$

$$g_{+Z-A\ominus+A-Z} = \alpha_{12}\frac{e^2}{s^2 c^2} ,$$

where all coefficients are defined in ref. [23]. We can also obtain these anomalous couplings from our theory by taking unitary gauge $\hat{U} = 1$. Matching these anomalous couplings from original EWCL with those obtained from our theory involving Z' boson, we obtain constraints which relate parameters in original EWCL with those in ours. These constraints

can be seen as an alternative result obtained through integrating out Z' field and its goldstone boson. Some of them are not independent each other and can be treated as self consistent check of our computation. Detailed matching for $M_{W^\pm}^2$ demands that the fundamental parameter f in (3.4) be the same as that introduced in original EWCL [23]. Matching for M_Z^2 gives

$$
\delta\beta_1 = g^2\alpha_3 + \frac{2\left(g^2+\Delta_g\right)}{\Delta_g{}^2}\left(g'\tilde{g}'\beta_2 - g''^2\beta_2^2\right) + \frac{2g^4 g'\tilde{g}'}{\Delta_g{}^2}\alpha_{24} - \frac{2g'\tilde{g}'\left[4g'^2 g''^2 - \left(g^2 - 4g''^2\right)^2\right]}{\Delta_g{}^2}\alpha_{25}
$$
$$
+\frac{4g'\tilde{g}'}{\Delta_g{}^3}\left[\left(4g''^2\Delta_g - g^2\Delta_g + 2g^2 g_Z^2\right)\beta_1\beta_2 - 4g''^2\left(2g^2 + \Delta_g\right)\beta_2\beta_3\right]\,, \tag{4.2}
$$

with $\delta\beta_1$ being the difference between β_1 introduced in (3.4) $\left.\beta_1\right|_{Z'}$ and corresponding parameter introduced in original EWCL $\left.\beta_1\right|_{\mathrm{EWCL}}$, i.e. $\delta\beta_1 = \left.\beta_1\right|_{Z'} - \left.\beta_1\right|_{\mathrm{EWCL}}$. While matching triple and quartic anomalous couplings gives

$$
\begin{aligned}
\delta\alpha_1 &= \delta\alpha_3 + \frac{2}{\Delta_g{}^2}\left(g'\tilde{g}'\beta_2 - g''^2\beta_2^2\right) + \frac{4g'\tilde{g}'}{\Delta_g{}^3}\left[\left(2g_Z^2 - \Delta_g\right)\beta_1\beta_2 - 8g''^2\beta_2\beta_3\right]\\
&\quad + \frac{2\tilde{g}'}{g'\Delta_g{}^2}\left\{\left[\left(g^2 - 4g''^2\right)^2 + 2g''^2\left(g^2 - 2g'^2\right)\right]\alpha_{24} + \left(g^2 - 4g''^2\right)\left(g'^2 + \Delta_g\right)\alpha_{25}\right\}\\
\delta\alpha_2 &= \frac{2}{\Delta_g{}^2}\left(g'\tilde{g}'\beta_2 - g''^2\beta_2^2\right) + \frac{4g'\tilde{g}'}{\Delta_g{}^3}\left[\left(2g_Z^2 - \Delta_g\right)\beta_1\beta_2 - 8g''^2\beta_2\beta_3\right]\\
&\quad + \frac{2\tilde{g}'}{g'\Delta_g{}^2}\left\{g^2 g'^2\alpha_{24} + \left(g^2 - 4g''^2\right)\left(g'^2 + \Delta_g\right)\alpha_{25}\right\}\\
\delta\alpha_4 &= \frac{4}{\Delta_g{}^2}\left(g'\tilde{g}'\beta_2 - g''^2\beta_2^2\right) + \frac{8g'\tilde{g}'}{\Delta_g{}^3}\left[\left(2g_Z^2 - \Delta_g\right)\beta_1\beta_2 - 8g''^2\beta_2\beta_3\right] + \frac{4g'\tilde{g}'}{\Delta_g{}^2}\left(g^2\alpha_{24} - g'^2\alpha_{25}\right)\\
&\quad - \frac{4g'\tilde{g}'}{\Delta_g}\alpha_{31}\\
\delta\alpha_5 &= -\delta\alpha_4\\
\delta\alpha_6 &= -\delta\alpha_4 - \frac{2g'\tilde{g}'}{\Delta_g}\alpha_{17}\\
\delta\alpha_7 &= \delta\alpha_4 - \frac{2g'\tilde{g}'}{\Delta_g}\alpha_{16}\\
\delta\alpha_8 &= -2\delta\alpha_3 - \frac{4}{\Delta_g{}^2}\left(g'\tilde{g}'\beta_2 - g''^2\beta_2^2\right) - \frac{8g'\tilde{g}'}{\Delta_g{}^3}\left[\left(2g_Z^2 - \Delta_g\right)\beta_1\beta_2 - 8g''^2\beta_2\beta_3\right]\\
&\quad + \frac{4g'\tilde{g}'}{\Delta_g{}^2}\left[\left(g'^2 - 4g''^2\right)\alpha_{24} + g'^2\alpha_{25}\right]\\
\delta\alpha_9 &= -2\delta\alpha_3 - \frac{4}{\Delta_g{}^2}\left(g'\tilde{g}'\beta_2 - g''^2\beta_2^2\right) - \frac{8g'\tilde{g}'}{\Delta_g{}^3}\left[\left(2g_Z^2 - \Delta_g\right)\beta_1\beta_2 - 8g''^2\beta_2\beta_3\right]\\
&\quad + \frac{2g'\tilde{g}'}{\Delta_g{}^2}\left[\left(\Delta_g - 2g^2\right)\alpha_{24} + 2g'^2\alpha_{25}\right] + \frac{2g'\tilde{g}'}{\Delta_g}\alpha_{31}\\
\delta\alpha_{10} &= -\frac{g'\tilde{g}'}{\Delta_g}\alpha_{15}\,, \tag{4.3}
\end{aligned}
$$

with $\delta\alpha_i = \alpha_i\big|_{Z'} - \alpha_i\big|_{\text{EWCL}}$ and left $\delta\alpha_3$ undetermined. In obtaining above result, we are accurate up to linear order of $\tilde{g}'$ and neglect all CP violation coefficients.

Beyond the self interaction part without Z' field, there is part depending on Z' field. The quadratic term is already discussed before and we list down the triple and quartic vertices,

$$\mathcal{L}_{Z'\text{anomalous}} = iC_{Z'-+}Z'^{\mu\nu}W_\mu^+W_\nu^- + iC_{+Z'-}(W_{\mu\nu}^+W^{-,\mu}Z'^\nu - W_{\mu\nu}^-W^{+,\mu}Z'^\nu) \qquad (4.4)$$
$$+D_{+-V_1V_2}W_\mu^+W^{-,\mu}V_{1,\nu}V_2{}^\nu+D_{+V_1-V_2}W_\mu^+V_1^\mu W_\nu^-V_2{}^\nu+D_{V_1V_2V_3V_4}V_{1,\mu}V_2{}^\mu V_{3,\nu}V_4{}^\nu.$$

The explicit expressions for various couplings in above Lagrangian are given in (A.3).

5. Summary

Stueckelberg mechanism as a traditional method to introduce a $\mathrm{U}(1)$ gauge boson into theory is shown in this paper equivalent to set up a gauged $\mathrm{U}(1)$ chiral Lagrangian and fix special gauge. With this equivalence to chiral Lagrangian, by constructing the non-abelian generalization of the chiral Lagrangian, it is easy to understand why non-abelian generalization of the Stueckelberg mechanism can not keep renormalizability. Further in terms of chiral Lagrangian formulation, we generalize traditional Stueckelberg mechanism by including in theory high dimension operators. We enlarge original EEWCL to include an extra local $\mathrm{U}(1)$ symmetry to represent physics for Z' boson. The scalar particle in Stueckelberg mechanism now is identified with goldstone boson eaten out by Z' to become its longitudinal component. We build up complete list of EEWCL up to order of p^4 including Z' and higgs bosons. With this chiral Lagrangian, traditional minimal version of the Stueckelberg mechanism can be seen as the leading nonlinear σ model term of our theory and our generalization for Stueckelberg mechanism is to include in theory all possible high dimension operators up to order of p^4. We obtain most general interaction for Z' boson and SM bosons. Among these interactions, we focus on the general mixing among neutral gauge boson W^3, B and X. We diagonalize the mixing appeared in mass and kinetic terms completely by introducing a three by three matrix U. The small mixing among X with W^3 and B can be achieved by constraints (3.27). Due to lack of enough theoretical constraints and experiment data, most of operators lead by our extension of Stueckelberg mechanism have their free couplings. We need to gather more theoretical arguments and experiment data to investigate them in future. Theoretically, through matching anomalous couplings between original EWCL and our theory, we obtain connections among parameters in ref. [23] and those in our theory which enable us to express anomalous couplings in terms of parameters appeared in our theory. We also exhibit all p^4 order operators for gauge fields self-interaction involving Z'.

Acknowledgments

A. Necessary formulae for EWCL

In this appendix, we list down the necessary lengthy formulae needed in the text. First we give expression for G_0 and G_1 introduced in (3.24),

$$
\begin{aligned}
G_0 = & -A_1 A_2 \Big\{ \left(-g^2 - g'^2 + g''^2 + (\tilde{g}')^2 \right) c_\alpha s_\alpha + g'\tilde{g}'\left(s_\alpha^2 - c_\alpha^2 \right) + g^2 \left[2g'^2 c_\alpha s_\alpha + g'\tilde{g}'\left(c_\alpha^2 - s_\alpha^2 \right) \right] \alpha_1 \\
& + g^2 \left[\left(g'^2 - g''^2 - (\tilde{g}')^2 \right) c_\alpha s_\alpha - g'\tilde{g}'\left(s_\alpha^2 - c_\alpha^2 \right) \right] \alpha_8 + 2g^2 g''^2 \left(c_\alpha^2 - s_\alpha^2 \right) \left(\alpha_{24} + g'^2 \alpha_1 \alpha_{25} \right) \\
& + g''^2 \left[-4g'\tilde{g}' c_\alpha s_\alpha + 2g'^2 \left(c_\alpha^2 - s_\alpha^2 \right) \right] \left[g^2 \left(\alpha_8 \alpha_{25} - \alpha_1 \alpha_{24} \right) - \alpha_{25} \right] + g^2 g''^2 \left[8g'^2 s_\alpha c_\alpha \right. \\
& \left. + 4g'\tilde{g}'\left(c_\alpha^2 - s_\alpha^2 \right) \right] \alpha_{24} \alpha_{25} + g^2 g'^2 g''^2 s_\alpha c_\alpha \left(4\alpha_{25}^2 - \alpha_1^2 \right) + 4g^2 g''^2 \left(g' s_\alpha + \tilde{g}' c_\alpha \right) \left(g' c_\alpha - \tilde{g}' s_\alpha \right) \alpha_{24}^2 \Big\} \\
G_2 = & A_1^2 \Big\{ (g^2 + g'^2) c_\alpha^2 + \left(g''^2 + (\tilde{g}')^2 \right) s_\alpha^2 \left(1 - g^2 \alpha_8 \right) - g^2 g'^2 c_\alpha^2 (2\alpha_1 + \alpha_8) + 4g'g''^2 \tilde{g}' s_\alpha^2 \alpha_{25} \\
& - 4g^2 g'^2 g''^2 c_\alpha^2 \left(\alpha_{24}^2 + \alpha_{25}^2 + 2\alpha_{24}\alpha_{25} \right) - g^2 g''^2 s_\alpha^2 \left[g'^2 \alpha_1^2 + 4(\tilde{g}')^2 \alpha_{24}^2 + 4g'\tilde{g}'\left(\alpha_8 \alpha_{25} - \alpha_1 \alpha_{24} \right) \right] \Big\} \\
& - \left[A_1 \to A_2, c_\alpha \leftrightarrow s_\alpha \right] + s_\alpha c_\alpha \left(A_1^2 + A_2^2 \right) \Big\{ -2g'\tilde{g}'\left[1 - g^2 \left(\alpha_1 + \alpha_8 \right) \right] \\
& + 4g^2 g''^2 \left[\left(\alpha_{24} - \alpha_{25} \right) \left(1 - g''^2 \alpha_1 \right) + 2g'\tilde{g}'\alpha_{24}^2 + g''^2 \alpha_8 \alpha_{25} \right] \Big\} .
\end{aligned} \tag{A.1}
$$

Next result is for rotation matrix U defined in (3.21), its matrix elements $U_{i,j}$ are

$$
\begin{aligned}
U_{1,1} = & \frac{g}{g_Z} \left[1 - \frac{g'^4}{g_Z^2} \alpha_1 + \frac{g^2(g^2 + 2g'^2)}{2g_Z^2} \alpha_8 \right] + \frac{2gg_Z}{\Delta_g^2} \left(g'\tilde{g}'\beta_2 - g''^2 \beta_2^2 \right) - 8\frac{g''^2 \tilde{g}' g'\left(g'^2 \Delta_g - g^2 g_Z^2 \right) g\alpha_{24}}{\Delta_g^2 g_Z^3} \\
& - 8\frac{g'^3 \tilde{g}' g''^2 \left(g_Z^2 + \Delta_g \right) g\alpha_{25}}{\Delta_g^2 g_Z^3} + \frac{4\tilde{g}' g' g_Z g}{\Delta_g^3} \left[4g''^2 \beta_2(\beta_1 - 2\beta_3) + g_Z^2 \beta_1 \beta_2 \right] \\
U_{1,2} = & \frac{g'}{g} U_{2,2} = \frac{g'}{g_Z} \left[1 + \frac{g^2 g'^2}{g_Z^2} \left(\alpha_1 + \frac{1}{2}\alpha_8 \right) \right] - \frac{2g^2 g'^2 \tilde{g}'}{g_Z^3} \left(\alpha_{24} + \alpha_{25} \right) \\
U_{1,3} = & \frac{2gg''}{\Delta_g} \left[\frac{g'\tilde{g}'}{2g''^2} - \beta_2 + \left(g'^2 - 4g''^2 \right) \alpha_{24} + g'^2 \alpha_{25} + 4g'\tilde{g}'(\beta_1 - \beta_3) - 2g_Z^2 \beta_1 \beta_2 + 8g''^2 \beta_2 \beta_3 \right] \\
& - \frac{2gg'\tilde{g}'(2g''^2 \Delta_g - g^2 g'^2)}{g''\Delta_g^2} \alpha_1 + \frac{g^3 g'\tilde{g}'(g'^2 - 4g''^2)}{g''\Delta_g^2} \alpha_8 + \frac{16gg'g''^2 \tilde{g}'}{\Delta_g^3} \left[g_Z^2 \beta_1^2 - \Delta_g \beta_1 \beta_3 + 4g''^2 \beta_3^2 \right] \\
& + \frac{2}{3}\frac{gg'g''\tilde{g}'\left(-112g_Z^2 g''^2 + 17g_Z^4 + 32g''^4 \right)}{g_Z^2 \Delta_g^3} \beta_2^2 \\
U_{2,1} = & -\frac{g'}{g_Z} \left[1 - \frac{g^4}{g_Z^2} \left(\alpha_1 + \frac{1}{2}\alpha_8 \right) \right] + \frac{2\tilde{g}'(g^2 - 4g''^2)g_Z}{\Delta_g^2} \beta_2 - 4\frac{\tilde{g}' g_Z \left(-g^2 \Delta_g + 8g'^2 g''^2 \right) \beta_1 \beta_2}{\Delta_g^3} \\
& + \frac{8g''^2 \tilde{g}'}{g_Z^3 \Delta_g^2} \left[g^2 \left(g^2 \Delta_g - g'^2 g_Z^2 \right) \alpha_{24} - 2g'^2 \left(g^2 \Delta_g - 2g'^2 g''^2 \right) \alpha_{25} \right] + \frac{2g''^2 g' g_Z \beta_2^2}{\Delta_g^2} \\
& - 16\frac{g''^2 \tilde{g}' g_Z \left(\Delta_g - 2g'^2 \right) \beta_2 \beta_3}{\Delta_g^3}
\end{aligned}
$$

$$U_{2,3} = \frac{2g'g''}{\Delta_g}\left(\frac{g^2-4g''^2}{2g'g''^2}\tilde{g}'+\beta_2+g^2\alpha_{24}+\left(g^2-4g''^2\right)\alpha_{25}\right)+\frac{g^2g'^2\tilde{g}'}{g''\Delta_g^2}\left[2\left(g^2-4g''^2\right)\alpha_1+g^2\alpha_8\right]$$

$$+\frac{4g'g''}{\Delta_g^2}\left(-2g'\tilde{g}'(\beta_1-\beta_3)+g_Z^2\beta_1\beta_2-4g''^2\beta_2\beta_3\right)+\frac{4g''g'}{\Delta_g^2}\left[g_Z^2\beta_1\beta_2-4g''^2\beta_2\beta_3\right]$$

$$-\frac{2}{3}\frac{g''\tilde{g}'\left(20g'^6-12g^4g''^2-136g^2g'^2g''^2-124g'^4g''^2+32g'^2g''^4+43g^2g'^4+26g^4g'^2+3g^6\right)}{g_Z^2\Delta_g^3}\beta_2^2$$

$$-\frac{16g'^2g''\tilde{g}'g_Z^2}{\Delta_g^2}\beta_1^2+\frac{16g'^2g''\tilde{g}'}{\Delta_g^3}\left[\left(2g_Z^2-\Delta_g\right)\beta_1\beta_3-4g''^2\beta_3^2\right]$$

$$U_{3,1} = \frac{2g''g_Z}{\Delta_g}\left(-\frac{2g'\tilde{g}'}{g_Z^2}+\beta_2+g^2\alpha_{24}-g'^2\alpha_{25}\right)+\frac{2g^2g'g''\tilde{g}'}{g_Z^3\Delta_g^2}\left[2\left(g^2\Delta_g-2g_Z^2g'^2\right)\alpha_1+g^2\left(2g_Z^2+\Delta_g\right)\alpha_8\right]$$

$$+\frac{4g''g_Z}{\Delta_g^2}\left(-2g'\tilde{g}'(\beta_1-\beta_3)+g_Z^2\beta_1\beta_2-4g''^2\beta_2\beta_3\right)-\frac{8}{3}\frac{g'\tilde{g}'g_Z\left(2g_Z^4-13g''^4-7g_Z^2g''^2\right)\beta_2^2}{g''\Delta_g^3}$$

$$-\frac{16g'g''\tilde{g}'g_Z}{\Delta_g^3}\left[g_Z^2\beta_1^2+4g''^2\beta_3^2+\left(\Delta_g-2g_Z^2\right)\beta_1\beta_3\right]$$

$$U_{3,2} = -\frac{g\tilde{g}'}{\sqrt{g_Z^2g''^2+g^2(\tilde{g}')^2-g^2g'^2g''^2\left(2\alpha_1+\alpha_8\right)+4g^2g'g''^3\tilde{g}'\left(\alpha_{24}+\alpha_{25}\right)}}$$

$$= -\frac{g\tilde{g}'}{g''g_Z}-\frac{g^3g'^2\tilde{g}'}{g''g_Z^3}\left(\alpha_1+\frac{1}{2}\alpha_8\right)$$

$$U_{3,3} = 1+\frac{2g''^2}{\Delta_g^2}\left[4g'\tilde{g}'\left(\beta_2+2\beta_2\beta_3+g^2\alpha_{24}-g'^2\alpha_{25}\right)-g_Z^2\beta_2^2\right]+\frac{32g'g''^2\tilde{g}'g_Z^2}{\Delta_g^3}\beta_2(\beta_1-\beta_3)\,, \quad \text{(A.2)}$$

where $g_Z = \sqrt{g^2+g'^2}$ and $\Delta_g = g^2+g'^2-4g''^2$ and except $U_{3,2}$ which vanishes when $\tilde{g}' = 0$, all other matrix elements are accurate up to linear order of $\tilde{g}'$.

The last formulae are the anomalous triple and quartic couplings for Z' field introduced in (4.4),

$$C_{Z'-+} = \frac{2g^2g''\left(\beta_2+g^2\alpha_{24}-g'^2\alpha_{25}\right)}{\Delta_g}+\frac{4g^2g''\left[g_Z^2\beta_1\beta_2-4g''^2\beta_2\beta_3\right]}{\Delta_g^2}-2\,g^2g''\alpha_{31}$$

$$-\frac{2}{3}\frac{g^2\tilde{g}'g'g''\left(17g_Z^4-112g_Z^2g''^2+32\,g''^4\right)\beta_2^2}{\Delta_g^3g_Z^2}+\frac{g^4g'\tilde{g}'}{\Delta_g^2g''}\left[\left(\Delta_g-2g'^2\right)\alpha_1+g^2\alpha_8\right]$$

$$-\frac{g^2g'\tilde{g}'}{g''\Delta_g}\left[1+g^2\left(\alpha_3+\alpha_9\right)+\left(g^2-4g''^2\right)\alpha_2\right]-8\frac{\tilde{g}'g'g^2g''(\beta_1-\beta_3)}{\Delta_g^2}$$

$$+\frac{16\tilde{g}'g'g^2g''}{\Delta_g^3}\left[\left(2g_Z^2-\Delta_g\right)\beta_1\beta_3-g_Z^2\beta_1^2-4g''^2\beta_3^2\right]$$

$$C_{+Z'-} = \frac{2g''g^2\left[\beta_2-\left(g'^2-4g''^2\right)\alpha_{24}-g'^2\alpha_{25}\right]}{\Delta_g}+\frac{4g^2g''\left[g_Z^2\beta_1\beta_2-4g''^2\beta_2\beta_3\right]}{\Delta_g^2}$$

$$-\frac{2}{3}\frac{g^2g'\tilde{g}'g''\left(17g_Z^4-112g_Z^2g''^2+32\,g''^4\right)\beta_2^2}{\Delta_g^3g_Z^2}-\frac{g^2g'\tilde{g}'}{\Delta_g g''}-4\frac{g'\tilde{g}'g^2g''\alpha_3}{\Delta_g}$$

$$-8\frac{g'\tilde{g}'g^2g''\left(\beta_1-\beta_3\right)}{{\Delta_g}^2}-\frac{g^2g'\tilde{g}'}{{\Delta_g}^2g''}\left[2\left(g^2g'^2-2g''^2\Delta_g\right)\alpha_1+g^2\left(g'^2-4g''^2\right)\alpha_8\right]$$

$$-16\,\frac{g'\tilde{g}'g^2g''}{{\Delta_g}^3}\left[g_Z^2{\beta_1}^2-\left(2g_Z^2-\Delta_g\right)\beta_1\beta_3+4g''^2\beta_3^2\right]$$

$$D_{+-Z'Z'}=4\,g^2g''^2\left(\alpha_5+\alpha_{21}\right)-\frac{4g^4g''^2{\beta_2}^2}{{\Delta_g}^2}+\frac{4g^4g'\tilde{g}'}{{\Delta_g}^2}\left[\beta_2-\left(g'^2-4g''^2\right)\alpha_{24}-g'^2\alpha_{25}\right]$$

$$-\frac{8g^2g'\tilde{g}'g''^2\alpha_{16}}{\Delta_g}+\frac{8g^4g'\tilde{g}'}{{\Delta_g}^3}\left[\left(2g_Z^2-\Delta_g\right)\beta_1\beta_2-8g''^2\beta_2\beta_3\right]$$

$$D_{+Z'-Z'}=4\,g^2g''^2\left(\alpha_4+\alpha_{19}\right)+4\frac{g^4g''^2{\beta_2}^2}{{\Delta_g}^2}-\frac{4g^4g'\tilde{g}'}{{\Delta_g}^2}\left[\beta_2-\left(g'^2-4g''^2\right)\alpha_{24}-g'^2\alpha_{25}\right]$$

$$-\frac{8g^2g'\tilde{g}'g''^2\alpha_{17}}{\Delta_g}-\frac{8g^4g'\tilde{g}'}{{\Delta_g}^3}\left[\left(2g_Z^2-\Delta_g\right)\beta_1\beta_2-8g''^2\beta_2\beta_3\right]$$

$$D_{+-ZZ'}=\frac{4g^4g''\left[\beta_2-\left(g'^2-4g''^2\right)\alpha_{24}-g'^2\alpha_{25}\right]}{g_Z\Delta_g}-2g^2g_Zg''\alpha_{16}+\frac{8g^4g''}{{\Delta_g}^2}\left(g_Z\beta_1\beta_2-4\frac{g''^2}{g_Z}\beta_2\beta_3\right)$$

$$-\frac{2g^4g'\tilde{g}'}{g_Zg''\Delta_g}\left[1+\left(2g_Z^2-\Delta_g\right)\alpha_3\right]-\frac{g'\tilde{g}'g^6\left(4g'^2\Delta_g-12g^2g''^2+g^2g_Z^2\right)\alpha_8}{g_Z^3\Delta_g^2g''}$$

$$-\frac{2g'\tilde{g}'g^4\left[g_Z^2\left(2g^2g'^2-g'^4+16g''^4\right)-4g^2g''^2(2g'^2+g'^2)\right]\alpha_1}{g_Z^3\Delta_g^2g''}$$

$$+8\,\frac{g^2g_Zg'\tilde{g}'g''}{\Delta_g}\left(\alpha_7-\alpha_{21}\right)-16\,\frac{g'\tilde{g}'g^4g''}{g_Z\Delta_g^2}\left(\beta_1-\beta_3\right)-128\,\frac{g'\tilde{g}'g^4g''^3{\beta_3}^2}{g_Z\Delta_g^3}$$

$$+32\frac{g^4g'\tilde{g}'g''}{g_Z\Delta_g^3}\left[\left(2g_Z^2-\Delta_g\right)\beta_1\beta_3+\frac{14g_Z^2g''^2-g_Z^4-4g''^4}{3}\beta_2^2\right]-32\frac{g^4g'\tilde{g}'g''g_Z{\beta_1}^2}{\Delta_g^3}$$

$$D_{+Z-Z'}=D_{+Z'-Z}=\frac{2g^4g''\left[-\beta_2+\left(g'^2-4g''^2\right)\alpha_{24}+g'^2\alpha_{25}\right]}{g_Z\Delta_g}-g^2g_Zg''\alpha_{17}$$

$$-\frac{4g^4g''}{\Delta_g^2}\left(g_Z\beta_1\beta_2-4\frac{g''^2}{g_Z}\beta_2\beta_3\right)+4\frac{g^2g_Zg'\tilde{g}'g''}{\Delta_g}(\alpha_6-\alpha_{19})+8\frac{g^4g'\tilde{g}'g''}{g_Z\Delta_g^2}(\beta_1-\beta_3)$$

$$+\frac{g^4g'\tilde{g}'}{g_Zg''\Delta_g}\left[1+\left(2g_Z^2-\Delta_g\right)\alpha_3\right]+16\frac{g^4g'\tilde{g}'g''g_Z{\beta_1}^2}{\Delta_g^3}$$

$$-16\frac{g^4g'\tilde{g}'g''}{g_Z\Delta_g^3}\left[\left(\imath g'^2+4g''^2+g^2\right)\beta_1\beta_3-4g''^2\beta_3^2\right]+\frac{g'\tilde{g}'g^6\left(4g'^2\Delta_g-12g^2g''^2+g^2g_Z^2\right)\alpha_8}{2g_Z^3\Delta_g^2g''}$$

$$+\frac{g'\tilde{g}'g^4\left(g^2g_Z^4-4g_Z^2g''^2\Delta_g-g'^4\Delta_g\right)\alpha_1}{g_Z^3\Delta_g^2g''}+\frac{16g'\tilde{g}'g^4g''\left(-14g_Z^2g''^2+4g''^4+g_Z^4\right)\beta_2^2}{3\Delta_g^3g_Z^3}$$

$$D_{+-AZ'}=\frac{4g^3g'g''\left[\beta_2-\left(g'^2-4g''^2\right)\alpha_{24}-g'^2\alpha_{25}\right]}{g_Z\Delta_g}-\frac{32g'g''g^3g''^2\beta_2\beta_3}{g_Z\Delta_g^2}+\frac{8g^3g'g''g_Z\beta_1\beta_2}{\Delta_g^2}$$

$$-\frac{2\tilde{g}'g^3g'^2}{g''g_Z\Delta_g}-\frac{128g''^3\tilde{g}'g^3g'^2\beta_3{}^2}{\Delta_g{}^3g_Z}-\frac{4g^3g'^2\tilde{g}'g''\left(17g_Z^4-112g_Z^2g''^2+32g''^4\right)\beta_2{}^2}{3g_Z^3\Delta_g{}^3}$$

$$+\frac{g'^2\tilde{g}'g^3}{\Delta_g{}^2g_Z^3g''}\left\{\left[8g_Z^2g''^2\Delta_g-g^2g'^2\left(4g_Z^2+2\Delta_g\right)\right]\alpha_1-g^2\left(3g'^2\Delta_g-8g^2g''^2\right)\alpha_8\right\}$$

$$-8\frac{g'^2\tilde{g}'g''g^3\alpha_3}{g_Z\Delta_g}-32\frac{\tilde{g}'g^3g''g'^2g_Z\beta_1{}^2}{\Delta_g{}^3}+32\frac{\left(2g_Z^2-\Delta_g\right)\tilde{g}'g^3g''g'^2\beta_1\beta_3}{\Delta_g{}^3g_Z}$$

$$-16\frac{g'^2\tilde{g}'g''g^3}{\Delta_g{}^2g_Z}\left(\beta_1-\beta_3+2\beta_2\beta_3\right)$$

$$D_{+A-Z'}=D_{+Z'-A}$$

$$=-\frac{2g^3g'g''\left[\beta_2-(g'^2-4g''^2)\alpha_{24}-g'^2\alpha_{25}\right]}{g_Z\Delta_g}+\frac{16g'g''g^3g''^2\beta_2\beta_3}{g_Z\Delta_g{}^2}-\frac{4g^3g'g''g_Z\beta_1\beta_2}{\Delta_g{}^2}$$

$$+\frac{8g'^2\tilde{g}'g''g^3}{g_Z\Delta_g{}^2}\left(\beta_1-\beta_3\right)-\frac{16g^3g''\tilde{g}'g'^2}{g_Z\Delta_g{}^3}\left[\left(2g_Z^2-\Delta_g\right)\beta_1\beta_3-4g''^2\beta_2\beta_3\right]$$

$$+\frac{2}{3}\frac{g'^2\tilde{g}'\left(17g_Z^4-112g_Z^2g''^2+32\,g''^4\right)g''g^3\beta_2{}^2}{g_Z^3\Delta_g{}^3}+\frac{16g^3g''g_Z\tilde{g}'g'^2\beta_1{}^2}{\Delta_g{}^3}$$

$$+\frac{4g'^2\tilde{g}'g^3g''\alpha_3}{g_Z\Delta_g}+\frac{g'^2\tilde{g}'g^3\left(2g^2g'^2g_Z^2-4g_Z^2g''^2\Delta_g+g'^2g^2\Delta_g\right)\alpha_1}{g''g_Z^3\Delta_g{}^2}$$

$$-\frac{1}{2}\frac{g^5g'^2\tilde{g}'\left(8\,g^2g''^2-3g'^2\Delta_g\right)\alpha_8}{g''g_Z^3\Delta_g{}^2}+\frac{g'^2\tilde{g}'g^3}{g''g_Z\Delta_g}$$

$$D_{Z'ZZZ}=-g_Z^3g''\left(2\alpha_{15}+\alpha_{16}+\alpha_{17}\right)$$

$$+4\frac{g'\tilde{g}'g_Z^3g''}{\Delta_g}\left(\alpha_6+\alpha_7+2\alpha_{10}-2\alpha_{18}-\alpha_{19}-2\alpha_{20}-\alpha_{21}\right)$$

$$D_{Z'Z'ZZ}=g''^2g_Z^2\left(\alpha_5+2\alpha_7+4\alpha_{20}+2\alpha_{21}\right)-4\frac{g''^2g'\tilde{g}'g_Z^2}{\Delta_g}\left(2\alpha_{15}+\alpha_{16}-2\alpha_{22}\right)$$

$$D_{Z'ZZ'Z}=4\,g''^2g_Z^2\left(\alpha_4+\alpha_6+2\alpha_{18}+\alpha_{19}\right)+32\frac{g''^2g'\tilde{g}'g_Z^2}{\Delta_g}\left(\alpha_{22}-\alpha_{15}\right)$$

$$D_{Z'Z'Z'Z}=-4\,g''^3g_Z\left(\alpha_{16}+\alpha_{17}+2\alpha_{22}\right)$$

$$+16\frac{g'\tilde{g}'g''^3g_Z}{\Delta_g}\left(\alpha_6+\alpha_7+2\alpha_{18}-\alpha_{19}+2\alpha_{20}-\alpha_{21}-4\alpha_{23}\right)$$

$$D_{Z'Z'Z'Z'}=4\,g''^4\left(\alpha_4+\alpha_5+2\alpha_{19}+2\alpha_{21}+4\alpha_{23}\right)-\frac{16g''^4g'\tilde{g}'}{\Delta_g}\left(\alpha_{16}+\alpha_{17}+2\alpha_{22}\right)\,. \quad \text{(A.3)}$$

References

[1] E.C.G. Stueckelberg, *Interaction energy in electrodynamics and in the field theory of nuclear forces*, *Helv. Phys. Acta* **11** (1938) 225.

[2] B. Körs and P. Nath, *A Stueckelberg extension of the standard model*, *Phys. Lett.* **B 586** (2004) 366 [hep-ph/0402047].

[3] D. Feldman, Z. Liu and P. Nath, *The Stueckelberg Z' at the LHC: discovery potential, signature spaces and model discrimination*, *JHEP* **11** (2006) 007 [hep-ph/0606294].

[4] B. Körs and P. Nath, *Aspects of the Stueckelberg extension*, *JHEP* **07** (2005) 069 [hep-ph/0503208].

[5] P. Ramond, *A pedestrian approach to covariant string theory*, *Prog. Theor. Phys. Suppl.* **86** (1986) 126.

[6] T. Gherghetta and A. Pomarol, *A Stueckelberg formalism for the gravitino from warped extra dimensions*, *Phys. Lett.* **B 536** (2002) 277 [hep-th/0203120].

[7] H. Ruegg and M. Ruiz-Altaba, *The Stueckelberg field*, *Int. J. Mod. Phys.* **A 19** (2004) 3265 [hep-th/0304245].

[8] T.J. Allen, M.J. Bowick and A. Lahiri, *Topological mass generation in $(3+1)$-dimensions*, *Mod. Phys. Lett.* **A 6** (1991) 559.

[9] M. Dittmar, A.-S. Nicollerat and A. Djouadi, *Z' studies at the LHC: an update*, *Phys. Lett.* **B 583** (2004) 111 [hep-ph/0307020].

[10] A. Leike, *The phenomenology of extra neutral gauge bosons*, *Phys. Rept.* **317** (1999) 143 [hep-ph/9805494].

[11] J.L. Hewett and T.G. Rizzo, *Low-energy phenomenology of superstring inspired E_6 models*, *Phys. Rept.* **183** (1989) 193.

[12] P.J. Franzini and F.J. Gilman, *Phenomenology of an extra neutral gauge boson in electron - positron collisions*, *Phys. Rev.* **D 35** (1987) 855.

[13] N. Arkani-Hamed, A.G. Cohen and H. Georgi, *Electroweak symmetry breaking from dimensional deconstruction*, *Phys. Lett.* **B 513** (2001) 232 [hep-ph/0105239].

[14] I. Antoniadis, *A possible new dimension at a few TeV*, *Phys. Lett.* **B 246** (1990) 377.

[15] T.G. Rizzo and J.D. Wells, *Electroweak precision measurements and collider probes of the standard model with large extra dimensions*, *Phys. Rev.* **D 61** (2000) 016007 [hep-ph/9906234].

[16] S. Weinberg, *Phenomenological Lagrangians*, *Physica* **A 96** (1979) 327.

[17] W. Buchmüller and D. Wyler, *Effective Lagrangian analysis of new interactions and flavor conservation*, *Nucl. Phys.* **B 268** (1986) 621.

[18] K. Hagiwara, T. Hatsukano, S. Ishihara and R. Szalapski, *Probing non-standard bosonic interactions via W-boson pair production at lepton colliders*, *Nucl. Phys.* **B 496** (1997) 66 [hep-ph/9612268].

[19] M.C. Gonzalez-Garcia, *Anomalous Higgs couplings*, *Int. J. Mod. Phys.* **A 14** (1999) 3121 [hep-ph/9902321].

[20] C. Arzt, M.B. Einhorn and J. Wudka, *Patterns of deviation from the standard model*, *Nucl. Phys.* **B 433** (1995) 41 [hep-ph/9405214].

[21] A.C. Longhitano, *Heavy Higgs bosons in the Weinberg-Salam model*, *Phys. Rev.* **D 22** (1980) 1166.

[22] T. Appelquist and C.W. Bernard, *Strongly interacting Higgs bosons*, *Phys. Rev.* **D 22** (1980) 200.

[23] T. Appelquist and G.-H. Wu, *The electroweak chiral Lagrangian and new precision measurements*, *Phys. Rev.* **D 48** (1993) 3235 [`hep-ph/9304240`].

[24] L.-M. Wang and Q. Wang, *Nonstandard Higgs in electroweak chiral Lagrangian*, `hep-ph/0605104`.

[25] D.A. Dicus and V.S. Mathur, *Upper bounds on the values of masses in unified gauge theories*, *Phys. Rev.* **D 7** (1973) 3111;
B.W. Lee, C. Quigg and H.B. Thacker, *Weak interactions at very high-energies: the role of the Higgs boson mass*, *Phys. Rev.* **D 16** (1977) 1519; *The strength of weak interactions at very high-energies and the Higgs boson mass*, *Phys. Rev. Lett.* **38** (1977) 883;
M.J.G. Veltman, *Second threshold in weak interactions*, *Acta Phys. Polon.* **B8** (1977) 475.

[26] C. Grosse-Knetter and R. Kogerler, *Unitary gauge, Stuckelberg formalism and gauge invariant models for effective Lagrangians*, *Phys. Rev.* **D 48** (1993) 2865 [`hep-ph/9212268`].

[27] Y. Zhang, S.-Z. Wang, F.-J. Ge and Q. Wang, *Electroweak chiral Lagrangian for left-right symmetric models*, *Phys. Lett.* **B 653** (2007) 259 [`arXiv:0704.2172`].

[28] J. Chay, K.Y. Lee and S.-h. Nam, *Bounds of the mass of Z' and the neutral mixing angles in general* $SU(2)_L \times SU(2)_R \times U(1)$ *models*, *Phys. Rev.* **D 61** (2000) 035002 [`hep-ph/9809298`].

[29] P.H. Frampton, M.B. Wise and B.D. Wright, *An elusive Z' coupled to beauty*, *Phys. Rev.* **D 54** (1996) 5820 [`hep-ph/9604260`].

[30] T.G. Rizzo, *Single charged Higgs boson production in Z decays*, *Phys. Rev.* **D 44** (1991) 3599.

[31] PARTICLE DATA GROUP collaboration, W.-M. Yao et al., *Review of particle physics*, *J. Phys.* **G 33** (2006) 1.

[32] B. Holdom, *Oblique electroweak corrections and an extra gauge boson*, *Phys. Lett.* **B 259** (1991) 329.

[33] B. Holdom, *Two* $U(1)$*'s and epsilon charge shifts*, *Phys. Lett.* **B 166** (1986) 196.

[34] T.G. Rizzo, *Gauge kinetic mixing and leptophobic Z' in E_6 and* $SO(10)$, *Phys. Rev.* **D 59** (1999) 015020 [`hep-ph/9806397`].

[35] D. Feldman, Z. Liu and P. Nath, *The Stueckelberg Z' extension with kinetic mixing and milli-charged dark matter from the hidden sector*, *Phys. Rev.* **D 75** (2007) 115001 [`hep-ph/0702123`].